EXERCISE – II
HINTS & SOLUTIONS

Sol.1 **A**

$x - 2y + 4 = 0$; $y^2 = 4x$; $\frac{x^2}{4} + \frac{y^2}{b^2} = 1$

$m = \frac{1}{2}$

C.O.T. For Both curves

$c = \frac{a}{m}$ & $c = \pm\sqrt{a^2m^2 + b^2}$

$\frac{a^2}{m^2} = A^2m^2 + b^2$

$\frac{1}{\frac{1}{4}} = 4 \times \frac{1}{4} + b^2 \Rightarrow b^2 = 4 - 1 \Rightarrow b = \sqrt{3}$

$c = \pm\sqrt{a^2m^2 + b^2}$

$c = \pm 2$

$c = \frac{a}{m}$

$m = \frac{a}{c} = \pm\frac{1}{2}$

$m = -\frac{1}{2}$

$y = mx + c$

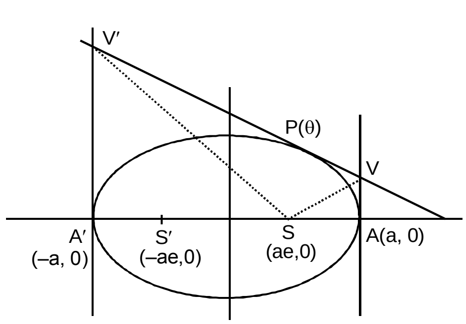
$y = -\frac{x}{2} - 2$

$2y + x + y = 0$

Sol.2 **A,C,D**

$P(a\cos\theta, b\sin\theta)$

Tangent at P



$\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta = 1$

$v\left(a, \frac{b(1-\cos\theta)}{\sin\theta}\right)$; $v'\left(-a, \frac{b(1+\cos\theta)}{\sin\theta}\right)$

$\ell(AV) = \frac{b(1-\cos\theta)}{\sin\theta}$; $\ell(A'V') = \frac{b(1+\cos\theta)}{\sin\theta}$

$\ell(AV).\ell(A'V') = \frac{b^2(1-\cos^2\theta)}{\sin^2\theta} = b^2$

$M_{SV} = \frac{b\left(\frac{1-\cos\theta}{\sin\theta}\right)}{a - ae}$; $M_{SV'} = \frac{b\left(\frac{1+\cos\theta}{\sin\theta}\right)}{-a - ae}$

$M_{SV} \times M_{SV'} = \frac{b^2}{-(a^2 - a^2e^2)} = \frac{b^2}{-b^2} = -1$

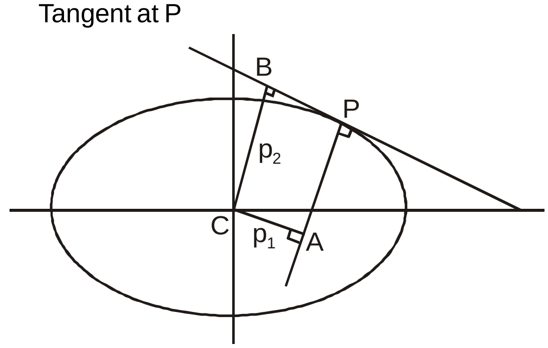
So $\angle VSV' =- 90^\circ$

So VS′V′S will be cyclic quadrilateral

Sol.3 **A**

$P\left(\frac{a}{\sqrt{2}}, \frac{b}{\sqrt{2}}\right)$

Tangent at P



$\frac{x}{\sqrt{2}a} + \frac{y}{\sqrt{2}b} = 1$ ....(1)

Normal at P ; $\sqrt{2}\,ax - \sqrt{2}\,by = a^2 - b^2$ ....(2)

$p_1 = \frac{\sqrt{2}ab}{\sqrt{a^2 + b^2}}$; $p_2 = \frac{a^2 - b^2}{\sqrt{2}\sqrt{a^2 + b^2}}$

So area of that rectangle OAPB = p_1p_2

$= \frac{(a^2 - b^2)\sqrt{2}ab}{\sqrt{2}(a^2 + b^2)} = \frac{ab(a^2 - b^2)}{a^2 + b^2}$

Sol.4 Given that:

$\frac{2b^2}{a} = a + b$

$2b^2 = a^2 + ab$

$b^2 - a^2 = ab - b^2$

$\Rightarrow (b - a)(b + a + b) = 0$

$b = a$

$\Rightarrow$ ellipse becomes a circle

Sol.5 C

$lx + my + n = 0$ ...(1) $|\alpha - \beta| = \frac{\pi}{2}$

$\frac{x}{2}\cos\left(\frac{\alpha+\beta}{2}\right) + \frac{y}{b}\sin\left(\frac{\alpha+\beta}{2}\right) = \cos\left(\frac{\alpha-\beta}{2}\right)$ ...(2)

Equation (1) and (2) are same line of chord

$$\frac{\cos\left(\frac{\alpha+\beta}{2}\right)}{a\ell} = \frac{\sin\left(\frac{\alpha+\beta}{2}\right)}{bm} = \frac{\cos\left(\frac{\alpha-\beta}{2}\right)}{-n} = \frac{-1}{\sqrt{2}n}$$

$$\cos\left(\frac{\alpha+\beta}{2}\right) = -\frac{a\ell}{\sqrt{2}n} \;;\; \sin\left(\frac{\alpha+\beta}{2}\right) = \frac{-bm}{\sqrt{2}n}$$

Square and add $\frac{a^2\ell^2}{2n^2} + \frac{b^2m^2}{2n^2} = 1$

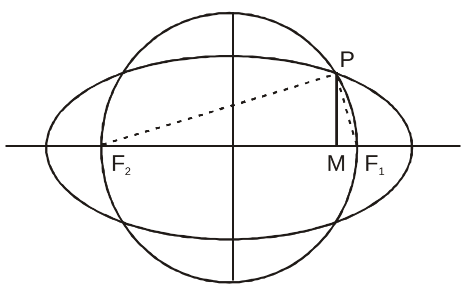
$a^2\ell^2 + b^2m^2 = 2n^2$

Sol.6 C

Equation of circle will be

$x^2 + y^2 = (ae)^2$ ....(1)

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ....(2)



$$\frac{(ae)^2 - y^2}{a^2} + \frac{y^2}{b^2} = 1$$

$y = \frac{b}{e}\sqrt{1-e^2} = \frac{a}{e}(1 - e^2)$ $2a = 17$

$PM = \frac{a}{e}(1 - e^2)$

Area of $\Delta PF_1F_2 = 30$

$\frac{1}{2}(F_1F_2) \times PM = 30$ $F_1F_2 = 2ae$

$\frac{1}{2}(2ae) \times \frac{a}{e}\sqrt{1-e^2} = 30$ $= 17 \times \frac{13}{17}$

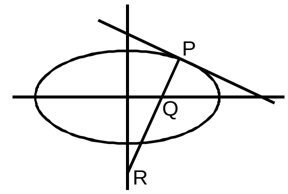
$e = \frac{13}{17}$ $F_1F_2 = 13$

Sol.7 C

Normal at $P(\theta)$

$ax\sec\theta - by\,\mathrm{cosec}\,\theta = a^2 - b^2$

$Q\left(\frac{a^2-b^2}{a\sec\theta}, 0\right)$



$R\left(0, \frac{a^2-b^2}{-b\,\mathrm{cosec}\theta}\right)$

Let the mid point of QR is M (h, k)

$2h = \left(\frac{a^2-b^2}{a}\right)\cos\theta \Rightarrow \cos\theta = \frac{2ha}{a^2-b^2}$

$2k = \left(\frac{a^2-b^2}{-b}\right)\sin\theta \Rightarrow \sin\theta = \frac{-2kb}{a^2-b^2}$

square & add

$$\frac{4h^2a^2}{(a^2-b^2)^2} + \frac{4k^2b^2}{(a^2-b^2)^2} = 1$$

$$\frac{4x^2a^2}{(a^2-b^2)^2} + \frac{4b^2y^2}{(a^2-b^2)^2} = 1$$

ecentricity $(e') = 1 - \dfrac{\frac{(a^2-b^2)^2}{4a^2}}{\frac{(a^2-b^2)^2}{4b^2}}$

$e' = 1 - \frac{b^2}{a^2} = e \Rightarrow e' = e$

Sol.8 C

Equation of normal at $P(x_1, y_1)$

$$\frac{a^2x}{x_1} - \frac{b^2y}{y_1} = a^2e^2$$

$T(x_1e^2, 0)$ $\frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} = 1$

$y_1^2 = \frac{b^2}{a^2}(a^2 - x_1^2)$

$PT = \sqrt{(x_1 - x_1e^2)^2 + y_1^2}$ $= (1 - e^2)(a^2 - x_1^2)$

$= \sqrt{x_1^2(1-e^2)^2 + y_1^2}$

$$= \frac{b}{a}\sqrt{a^2 - x_1^2 e^2}$$

$$= \frac{b}{a}\sqrt{rr_1} \qquad r = a + ex_1 \;;\; r_1 = a - ex_1$$

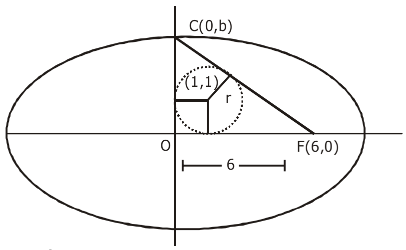
Sol.9 **A**

Eq^n of CF:

$$\frac{x}{6} + \frac{y}{b} = 1$$

p = r

$$\left|\frac{\frac{1}{6} + \frac{1}{b} - 1}{\sqrt{\frac{1}{36} + \frac{1}{b^2}}}\right| = 1$$



$\Rightarrow b = 5/2 \Rightarrow 2b = 5$

ae = 6

$e^2 = 1 - b^2/a^2 \Rightarrow a^2e^2 = a^2 - b^2 \Rightarrow 36 = a^2 - 25/4$

$\Rightarrow a^2 = 169/4 \Rightarrow a = \dfrac{13}{2}$

$2a = 13 \Rightarrow$ (AB) (CD) = 5 × 3 = 65

Sol.10 **A,B,C**

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

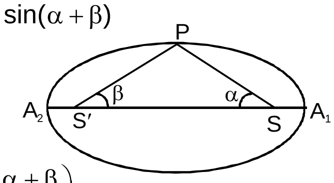
$$\left.\begin{array}{l}\text{If } a > b \;;\; SP + S'P = 2a \\ \text{If } a < b \;;\; SP + S'P = 2b\end{array}\right\}\text{By property}$$

In $\Delta PSS'$ apply sine rule

$$\frac{SP}{\sin\beta} = \frac{S'P}{\sin\alpha} = \frac{SS'}{\sin[\pi - (\alpha + \beta)]}$$

or $\dfrac{SP + S'P}{\sin\alpha + \sin\beta} = \dfrac{2ae}{\sin(\alpha + \beta)}$

$$\frac{\sin(\alpha + \beta)}{\sin\alpha + \sin\beta} = \frac{e}{1}$$



$$\frac{2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)}{2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)} = \frac{e}{1}$$

$$\frac{e}{1} = \frac{\cos\left(\frac{\alpha+\beta}{2}\right)}{\cos\left(\frac{\alpha-\beta}{2}\right)}$$

$$\frac{1-e}{1+e} = \frac{2\sin\frac{\alpha}{2}\sin\frac{\beta}{2}}{2\cos\frac{\alpha}{2}\cos\frac{\beta}{2}}$$

$$\frac{1-e}{1+e} = \tan\frac{\alpha}{2}\tan\frac{\beta}{2}$$

Sol.11 **A,B**

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

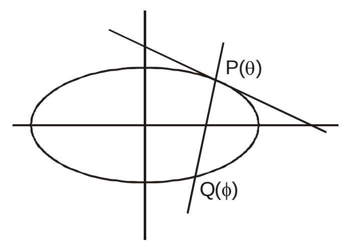
chord PQ :

$$\frac{x}{a}\cos\left(\frac{\theta+\phi}{2}\right) + \frac{y}{b}\sin\left(\frac{\theta+\phi}{2}\right) = \cos\left(\frac{\theta-\phi}{2}\right)$$

If it is passes through point (d, 0) on axis

$$\frac{d}{a}\cos\left(\frac{\theta+\phi}{2}\right) = \cos\left(\frac{\theta-\phi}{2}\right)$$

$$\frac{d}{a} = \frac{\cos\left(\frac{\theta-\phi}{2}\right)}{\cos\left(\frac{\theta+\phi}{2}\right)}$$



C & D

$$\frac{d-a}{d+a} = \frac{\cos\left(\frac{\theta-\phi}{2}\right) - \cos\left(\frac{\theta+\phi}{2}\right)}{\cos\left(\frac{\theta-\phi}{2}\right) + \cos\left(\frac{\theta+\phi}{2}\right)}$$

$$= \frac{2\sin\frac{\theta}{2}\sin\frac{\phi}{2}}{2\cos\frac{\theta}{2}\cos\frac{\phi}{2}}$$

$$\tan\frac{\theta}{2}\tan\frac{\phi}{2} = \frac{d-a}{d+a}$$

$$d = ae \Rightarrow \tan\frac{\theta}{2}\tan\frac{\phi}{2} = \frac{ae-a}{ae+a} = \frac{e-1}{e+1}$$

$$d = -ae \Rightarrow \tan\frac{\theta}{2}\tan\frac{\phi}{2} = \frac{-ae-a}{-ae+a} = \frac{e+1}{e-1}$$